**Last revised on January 31 2012.**

## The unified equation applied to API 15HR

**Introduction** – This exercise illustrates the application of the unified equation to the long-term cyclic requirement in API 15HR. The requirement is "no pipe failure after exposure to N = $10^9$ cycles at nominal pressure and a strain ratio R = 0.9". The problem will be addressed from two perspectives, namely burst failure from fiber rupture and weep failure from resin (interphase) rupture.

**The global strains** – We start with an estimation of the strains in the global "x" (axial) and "y" (hoop) directions of the pipe. From these we calculate the strains in the principal directions of the UD plies, that is, the strains transverse to the fibers (direction 2) and in the fiber direction (direction 1).

The strain transverse to the fiber controls resin (interphase) rupture and the weep failure. The strain in the fiber direction controls fiber rupture and the burst failure. The calculations assume the pipe operating under nominal conditions, as required by API 15HR.

We are interested in the relationship between the hoop and axial global strains for angle-ply ± 55 laminates (70% glass loading) under a 2:1 pressure loading. This relationship is derived like follows:

The global stresses in cylinders under internal pressure are related to the global strains by the following relationship.

$$[\sigma] = [A] \times [\varepsilon]$$

The stiffness matrix [A] for ± 55 angle-ply laminates with 70% glass loading is

$$[A] = \begin{bmatrix} 132470 & 92200 & 0 \\ 92200 & 235070 & 0 \\ 0 & 0 & 101220 \end{bmatrix}$$

Entering this in the above equation we have

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} 132470 & 92200 & 0 \\ 92200 & 235070 & 0 \\ 0 & 0 & 101220 \end{bmatrix} \times \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} \qquad (1)$$

Assuming a 2:1 pressure loading and no external torque, equation (1) reduces to

$$\begin{bmatrix} \sigma_x \\ 2\sigma_x \\ 0 \end{bmatrix} = \begin{bmatrix} 132470 & 92200 & 0 \\ 92200 & 235070 & 0 \\ 0 & 0 & 101220 \end{bmatrix} \times \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} \qquad (2)$$

From the above we have the relationships

$$\varepsilon_y = (3.41)\varepsilon_x \quad (3A)$$

$$\gamma_{xy} = 0 \quad (3B)$$

Equation (3A) indicates that in ± 55 angle-ply pipes under 2:1 pressure loading the hoop strains are 3.41 times the axial strains. Equation (3B) indicates absence of shear strains in the global reference frame.

The long-term (20 years) hoop strain per API 15HR is obtained from the weep regression equation for static loading at 65C. We use the regression line from a well known pipe manufacturer.

$$\log(LTHS) = -0.01 - 0.06\log(20\times 365\times 24)$$

$$LTHS = 0.48\%$$

The above static long-term hoop strain – LTHS – weeps the pipe in 20 years of continuous operation. It should be interpreted like follows: "The pipes subjected to a hoop static strain of 0.48% develops small cracks that allow the passage of water. It would take the water 20 years to travel through the cracked pipe wall". The regression equation does not imply any deterioration of the pipe. It simply measures the time taken by the water to travel through the cracked wall.

From API 15HR the allowable static long-term hoop strain is

$\varepsilon_y = f_1 f_2 (LTHS) = 0.85\times 0.67\times 0.48 = 0.27\%$ (Allowable static hoop strain for a weep life of 20 years)

Entering the above in equation (3) we obtain the static axial strain

$$\varepsilon_x = \frac{0.27}{3.41} = 0.08\%$$ (Static axial strain for a weep life of 20 years)

**The strains in the local system –** The strains in the principal ply directions 1 – 2 are obtained by rotating the strains in the global system x – y. The rotation matrix for ± 55 plies is

$$[T] = \begin{bmatrix} 0.33 & 0.67 & \pm 0.94 \\ 0.67 & 0.33 & \mp 0.94 \\ \mp 0.47 & \pm 0.47 & -0.34 \end{bmatrix}$$

The long-term static strains in the principal directions 1 – 2 are

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \frac{1}{2}\gamma_{12} \end{bmatrix} = \begin{bmatrix} 0.33 & 0.67 & \pm 0.94 \\ 0.67 & 0.33 & \mp 0.94 \\ \mp 0.47 & \pm 0.47 & -0.34 \end{bmatrix} \times \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} \quad (4)$$

Entering the global strains in the above we have

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \frac{1}{2}\gamma_{12} \end{bmatrix} = \begin{bmatrix} 0.33 & 0.67 & \pm 0.94 \\ 0.67 & 0.33 & \mp 0.94 \\ \mp 0.47 & \pm 0.47 & -0.34 \end{bmatrix} \times \begin{bmatrix} 0.08 \\ 0.27 \\ 0 \end{bmatrix}$$

$\varepsilon_1 = 0.18\%$ (Static strain in the fiber direction for a weep life of 20 years)

$\varepsilon_2 = 0.14\%$ (Static strain in the transverse direction for a weep life of 20 years)

$\gamma_{12} = \pm 0.18\%$ (Shear strain referred to the ply system)

The above are the static strains acting on the principal directions 1 – 2 of the ± 55 UD plies under the API 15HR operating conditions. They were calculated assuming the pipe operating under rated pressure.

**Burst failure** – The burst failure is controlled by the rupture of the glass fibers. The unified equation predicts the long-term burst failure of pipes operating under the simultaneous action of static and cyclic tensile loads. The unified equation for strains acting in the ply (fiber) direction 1 is

$$\left(\frac{\varepsilon \times SF}{S_S}\right)^{\frac{1}{G_S}} + \left(\frac{\Delta\varepsilon \times SF}{S_C}\right)^{\frac{1}{G_C}} + \left(\frac{\varepsilon \times \Delta\varepsilon \times SF^2}{S_S \times S_C}\right)^{\frac{1}{G_{SC}}} = 1{,}0 \qquad (5)$$

Where

$\epsilon = \epsilon_1 = 0.18\%$ is the tensile static strain in the fiber direction (direction 1)
$\Delta\epsilon = \Delta\epsilon_1 = 0.018\%$ (for R = 0.9) is the cyclic tensile strain range in the fiber direction
Ss = is the long-term (20 years) static strength of the glass fibers in the 1 direction
Sc = is the long-term cyclic strength of the glass fibers in the 1 direction
Gs = 0.077 (Mark Greenwood, boron-free glass)
Gs = 0.130 (Mark Greenwood, E glass)
Gc = 0.089 (John Mandell and Guangxu Wei, any glass)
Gsc = 15 (Antonio Carvalho, unpublished)

The long-term static and cyclic strengths for the UD plies in the fiber direction are obtained from the appropriate regression lines.

*The long-term static strengths in the 1 direction are*

$\log(S_S) = 0.400 - 0.077\log(20 \times 365 \times 24)$ (Mark Greenwood for boron-free glass)

$S_S = 0.99\%$ (for boron-free glass)

$\log(S_S) = 0.347 - 0.130\log(20 \times 365 \times 24)$ (Mark Greenwood for E glass)

$S_S = 0.46\%$ (for E glass)

*The long-term cyclic strength in the 1 direction is*

$$\log(S_C) = 0.519 - 0.089\log(N)$$

Where, according to API 15HR, the number of cycles in 20 years is N = $10^9$.

$$\log(S_C) = 0.519 - 0.089\log(10^9)$$

The long-term cyclic strength in the 1 direction is

$S_C = 0.522\%$ (Long-term cyclic strength for both boron-free and E glasses)

Entering these values in the unified equation we have

**For pipes made of boron-free glass:**

$$\left(\frac{0.18\times SF}{0.99}\right)^{\frac{1}{0.077}} + \left(\frac{0.018\times SF}{0{,}522}\right)^{\frac{1}{0.089}} + \left(\frac{0.18\times 0.018\times SF^2}{0.99\times 0{,}522}\right)^{\frac{1}{15}} = 1.0$$

SF = 4.6

The Unified Equation predicts that pipes made of boron-free glass under the combined action of cyclic and static loads as mandated in API 15HR have a long-term (20 years) safety factor against burst rupture equal to SF = 4.6

**For pipes made of E glass**:

$$\left(\frac{0.18\times SF}{0.46}\right)^{\frac{1}{0.130}} + \left(\frac{0.018\times SF}{0{,}522}\right)^{\frac{1}{0.089}} + \left(\frac{0.18\times 0.018\times SF^2}{0.46\times 0{,}522}\right)^{\frac{1}{15}} = 1.0$$

SF = 2.0

The lower hydrolytic stability of E glass (compared to boron-free glass) results in a lower long-term (20 years) safety factor against burst rupture SF = 2.0.

We conclude that pipes made of either boron-free or regular E glass meet the API 15HR requirements against burst failure under the combined action of static + cyclic loading. Also, the analysis establishes the superior performance of boron-free glass versus regular E glass.

The above analysis is valid for long-term burst rupture. We next address the weep mode of failure.

**Weep failure –** The weep mode of failure results from water seeping through cracks that form in the fiber-resin interphase. The cracks form and grow in the interphase, avoiding the resin matrix. There is no doubt that the transverse cracks in UD plies result from glass-resin debonding as well as by cracking of the interphase. Weeping results from the passage of water through these cracks.

The opening of these transverse cracks is controlled by the magnitude of the tensile strain and is usually very small. The cracks that form from static tensile loads are stationary, i.e., they do not grow. The cracks that form from cyclic tensile loads increase in length and density (number of cracks per unit volume) but do not increase in opening. The length and density of the cracks under cyclic loads depends on the number of cycles. It is important that we understand this. The crack opening is determined by the strain level, while the crack length is determined mostly by the number of cycles in cyclic loadings. The underlying reason for this is that cyclic loads grow cracks, while static loads do not. As a rule, the transverse cracks that coalesce to form the pathway that leads to weep failure, are wider and shorter under static loads, than they are under cyclic loads.

In the process of weeping, the water molecules move along many narrow and long cracks. Starting from the inner surface, the molecules move along cracks in the + 55 ply until they cross with similar cracks in the – 55 ply immediately above it. At this crossing point the molecules migrate from the inner to the outer ply and proceeds this way until they weep out. The cracks are narrow, the paths are tortuous and the travel distances are large. The time taken by the water to move from the inner ply to the outer ply may be very long. It is obvious that the weep time depends on the pipe thickness. Other things being equal, the pipe with larger wall thickness will have longer weep time.

We see that the weep time is not a fundamental material property, since it depends on the pipe thickness. One very simple way to obtain high LTHS in the ASTM D 2992 test is to increase the wall thickness of the test specimens. The LTHS (or HDB) is therefore not a fundamental material property and should not be used in pipe design. The fundamental pipe parameter controlling the weep process is the transverse strain that coalesces to form the required pathway. This fundamental parameter is known as threshold strain

The threshold strain depends on the glass-resin bonding as well as on the toughness of the interphase. The threshold strain has nothing to do with fiber rupture or with the composition of the glass fibers.

*Note: The properties of the interphase depend on the base resin as well as on the fiber sizing. Over the years, the sizing chemistry has been optimized for use in epoxy resins. A similar effort is required for polyesters and vinyl esters.*

The threshold strain is a “safe strain” for weeping from static loads. Under small static loads, the cracks are too few or too short and do not coalesce to form the pathway for the water. As it happens, the threshold strain is not a safe strain for weeping under cyclic loads. Under cyclic loads the cracks will grow and eventually coalesce even if their opening is less than that of the static threshold strain. According to Guangxu Wei in his master thesis published by the Montana State University, the threshold strain for tensile loads in the 2 direction of polyester UD plies is 0.25%. Acoustic emission studies conducted at the University of Liverpool have suggested a threshold strain value of 0.20%. In this paper we will use 0.20% as the threshold strain in the transverse direction of polyester UD plies.

Returning to our problem, we note that the safety factor against the peak strain exceeding the threshold strain in cases of cyclic and static loadings is

$$SF = \frac{(threshold \quad strain)}{\varepsilon + \frac{\Delta\varepsilon}{2}} \tag{6}$$

In API 15HR pipes operating under rated conditions, the static strain in the 2 direction was calculated earlier as $\epsilon_2$ = 0.14%. The cyclic component of the tensile strain in the 2 direction, for R = 0.9, is $\Delta\epsilon_2$ = 0.014%. Therefore, the safety factor against peak strains is

$$SF = \frac{0,20}{0,14 + \frac{0,014}{2}} = 1,3 \tag{7}$$

We see that there is no risk of the pipe ever weeping under the peak strain resulting from the combined action of static and cyclic strains specified in API 15HR. So, we are safe against weep failure as far as the magnitude of the combined loading is concerned.

But this is a cyclic loading that grows cracks and eventually fail the pipe. We next check the pipe for cyclic weep failure. Weep failure under cyclic loads requires the fulfillment of two conditions. First, the initial small cracks must grow under the cyclic strain until they coalesce to form the pathway. Let us call this the crack time. And second, the water must travel the pathway thus formed. Let us call this the travel time. Therefore, the weep time under cyclic loading is composed of two times.

$$[weep \quad time] = [crack \quad time] + [travel \quad time]$$

We next use the unified equation to calculate the crack time. For transverse tensile rupture in the 2 direction the unified equation is

$$\left(\frac{\Delta\varepsilon \times SF}{S_C}\right)^{\frac{1}{0,040}} + \left(\frac{\varepsilon \times \Delta\varepsilon \times SF^2}{S_S \times S_C}\right)^{\frac{1}{G_{SC}}} = 1,0 \tag{8}$$

Where

$\Delta\epsilon$ = 0.014% (Cyclic component in the 2 direction. R = 0.9)
$S_c$ = 0.087% (N = $10^9$ cycles)
$S_s$ = 0.20% (Short-term strength in the 2 direction)
$\epsilon$ = 0.14% (Static component in the 2 direction)
$G_{sc}$ = ? (Unknown for R = 0.9)

*Note: The long-term cyclic strength $S_c$ =0.087% was determined from the Guangxu Wei cyclic regression equation for UD plies tensioned in the 2 direction for N = $10^9$ cycles*

$$\log(S_c) = \log 0{,}20\% - 0{,}040 \log N$$

$$\log(S_c) = \log 0{,}20\% - 0{,}040 \times \log 10^9$$

*$S_c$ = 0.087%*

Entering the above in the unified equation we obtain

$$\left(\frac{0{,}014 \times SF}{0{,}087}\right)^{\frac{1}{0{,}040}} + \left(\frac{0{,}14 \times 0{,}014 \times SF^2}{0{,}20 \times 0{,}087}\right)^{\frac{1}{G_{SC}}} = 1{,}0 \qquad (9)$$

The unified equation cannot be used to compute the safety factor SF for the ply rupture in this case, because the interaction parameter $G_{sc}$ is not known for this particular loading (R = 0.9). We can, however, solve equation (9) using the conservative value $G_{sc}$ = 60 000 which is valid for R = 0.5.

Entering $G_{sc}$ = 60000 in equation (9) we obtain

SF = 2.5

The safety factor SF = 2.5 indicates that the UD plies will not crack under the specified loading in 20 years. We can be sure that for a desired lifetime of 20 years the pipes under the cyclic loading prescribed in API 15HR will not crack and therefore not weep.

This complete our analysis of the API 15HR cyclic problem.

**Conclusion –** The analysis conducted using the unified equation reveals that:

1. The long-term cyclic requirement imposed by API 15 HR (N = $10^9$ cycles in 20 years at R = 0.9) is easily met for rupture (burst) failure.
2. The boron-free glass is superior to the regular E glass to sustain rupture failure.
3. The unified equation could not be used to do the weep analysis for lack of specific data to estimate the interaction coefficient Gsc for R = 0.9 and N = $10^9$
4. However, using conservative data for R = 0.5 and N = $10^9$, we have determined that the pipes will not weep in 20 years under the combined static + cyclic loading specified in API 15HR.